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Challenges in Biochemical Engineering and Biotechnology for Sustainable Environment

Design of an Intelligent Control Scheme for Synchronizing Two Coupled Van Der Pol Oscillators

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Abstract: A well known common feature of oscillatory system and biological oscillator, in particular, is their ability to synchronize. Synchronization can be defined as, the adjustment of rhythms of oscillating objects due to their weak interaction. The phenomenon which results in the matching of the output frequency with the input frequency is called frequency entrainment or synchronization. An Oscillator is any system that exhibits periodic behaviour. Synchrony is the most familiar mode of organization for coupled oscillators but when two or more oscillators are coupled together, the ranges of possible behaviour become much more complex. As the qualitative features of the excitation potential of the heart is very close to the dynamic behaviour of the classical oscillator Van der Pol, the same is considered in this work. The Van der Pol oscillator is an oscillator with nonlinear damping, governed by the second order differential equation, $\ddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = 0$, where '$x$' is the position coordinate, a dynamic variable, and 'ε' is a scalar parameter which controls the nonlinearity and the strength of the damping.

Two such Van der pol Oscillators are bidirectionally coupled where the coupling factor induces an adjustment of the rhythms on to a common synchronized manifold, thereby inducing mutual synchronization behavior. Suitable variations are induced in the parameters x and ε for the coupled oscillators and the phenomena of synchronization among them is studied using MatLab software for which the coupled pair fail to synchronize. In order to establish mutual consensus between the two coupled oscillators that are otherwise not in synchrony, five different control strategies have been designed and the performance measures of the implemented control schemes are also evaluated. Out of the various control schemes to entrain two Van der pol oscillators to synchronize over time, the simulation results quantify that the proposed intelligent Fuzzy Logic Control (FLC) scheme gives optimal performance ahead of the rest.

Keywords: Synchronization, Entrainment, Oscillator, Coupling, Control.

1. Introduction

Biological oscillators are amenable to qualitative analysis even before they have been described exhaustively in quantitative terms. Qualitative analysis can identify the elements essential for generating the oscillations and can enhance ones understanding of underlying oscillator mechanisms [1]. The essential elements of a biological oscillator includes an inhibitory feedback loop with one or more oscillating variables, and a source of delay in the feedback loop, which allows an oscillating variable to overshoot a steady-state value before the feedback inhibition is wholly effective. The analysis of the patterns of interactions and delays observed in biological oscillators is simplified by the translation of variables, interactions, and delays into schematic representations. Mathematical models of nonlinear oscillators are used to describe a wide variety of physical and biological phenomena that exhibit self-sustained oscillatory behaviour [2]. When these oscillators are strongly driven by forces that are periodic in time, they often exhibit a remarkable ‘‘mode-locking’’ phenomena that synchronizes the nonlinear oscillations to the driving force.

2. Synchronization

A well known common feature of oscillatory system and biological oscillator, in particular, is their ability to synchronize. Synchronization is bringing of two or several processes into a state of synchrony, that is, into a state such that identical or corresponding elements of the processes occur at a fixed phase difference relative to one another. Synchronization can also be defined as, the adjustment of rhythms of oscillating objects due to their weak interaction [3]. Synchronization between dynamical systems and analysis of synchronization phenomena in the evolution of dynamical systems has been an active field of research in many scientific and technical disciplines. Multiple periodic processes with different natural frequencies come to acquire a common frequency and in some cases also a common phase as a result of their mutual influence. In fact interaction between individual oscillators may lead to mutual entrainment of their cycles and thus to emergence of coherent internal dynamics. The output frequency at some point exactly matches with the input frequency and continue to remain as such thereafter. The phenomenon which results in the matching of the output frequency with the input frequency is called frequency entrainment or synchronization [4].

3. Nonlinear Oscillators

Active oscillators can generate spontaneous oscillations, which continue indefinitely. This is in contrast to passive oscillators such as the strings of a piano, where oscillations die down. In the case of spontaneous oscillations, a variable of the system changes periodically in time. The shape of oscillations can be sinusoidal or follow a different periodic function. Oscillations are characterized by their amplitude and phase. The amplitude is the maximal value the variable attains during a period; the phase indicates the state of the oscillator relative to the beginning of a period. Spontaneous oscillations occur only in nonlinear dynamic systems that are open (i.e., there is a continuous flow of energy through the system from its environment) [5]. Biological systems fall in this category and in general are able to exhibit oscillations. Relaxation oscillations are recognized as having similarities to biological oscillations. A very old and important result in nonlinear dynamics is that when nonlinear oscillators with stable limit cycles are subject to periodic perturbations, they may become entrained to the perturbation. Rhythmic variations in blood pressure, heart pulse and other cardiovascular measures indicate importance of understanding the dynamic aspects of cardiovascular rhythms. Cardiac conduction system can be considered as a network of elements that self stimulates, such as: Sino Atrial (SA) node (the first pacemaker), Atrio Ventricular (AV) node and His Purkinje fibre system. For the reason that these elements show oscillation behaviour, they can be modelled as nonlinear oscillators [4].

4. Van Der Pol Oscillator

Conductive system of the heart can be stimulated to act as a network of elements and these elements shows the oscillatory behaviour that can be modelled as nonlinear oscillators. Since the qualitative features of the excitation potential of the heart is very close to the dynamic behaviour of the classical oscillator Van der Pol, this oscillator can be considered as the starting point for this modelling [4]. The differential equation $\ddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = 0,$ is called the van der Pol oscillator, where x is the position coordinate which is a dynamical variable, and $\varepsilon > 0$ is a scalar parameter which controls the nonlinearity and the strength of the damping. The state variables are assumed for the above differential equation as $x1= x$ and $x2=\dot{x}$. The dynamic behaviour of the oscillator model using the phase-plane method is initially studied. Phase plane analysis is an important technique used for studying the behaviour of non linear systems, since there is no analytical solution for a nonlinear system. The limit cycle describes the oscillations of nonlinear system which is a closed trajectory in phase space having the property that at least one other trajectory spirals into it either as time approaches infinity or as time approaches negative infinity. Such behaviour is exhibited in Van der Pol oscillators that belong to the category of nonlinear systems [6].

5. Limit Cycle

A system to approach a periodic behaviour, which will thus, appears a closed curve in phase plane is called a Limit Cycle [7]. It is having the property that at least one other trajectory spirals into it either as time approaches infinity or as time approaches negative infinity. Such behaviour is exhibited in some nonlinear systems. This includes the possibility that a distributed nonlinear system even while staying within the tolerance limits, may exhibit a special behaviour of following a closed trajectory or limit cycle which describes the oscillations of nonlinear system [8]. The existence of a limit cycle corresponds to an oscillation of fixed amplitude and period. The limit cycle obtained is stable, since the trajectory converges towards it. The limit cycle representation in phase plane for the model is carried out, where $x1$ and $x2$ corresponds to the two states of the van der pol oscillator.

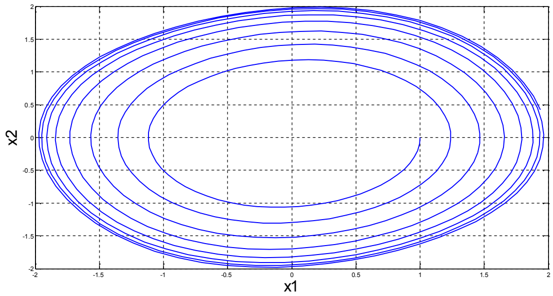

Fig 5.1. Limit cycle representation in Phase plane, For $x1$(0)=1 , $x2$(0)=0 , ε=0.1.

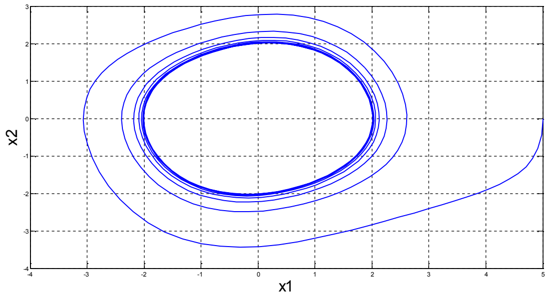

Fig 5.2. Limit cycle representation in Phase plane, For $x1$(0)=5 , $x2$(0)=0 , ε=0.1.

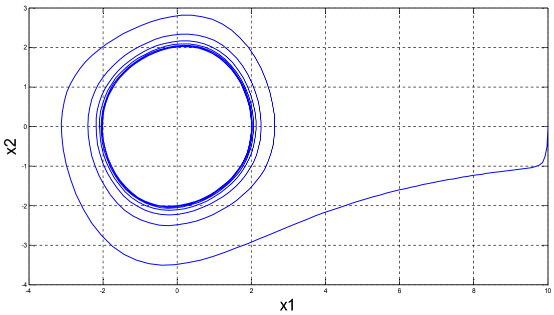

Fig 5.3. Limit cycle representation in Phase plane, For $x1$(0)=10, $x2$(0)=0 , ε=0.1.

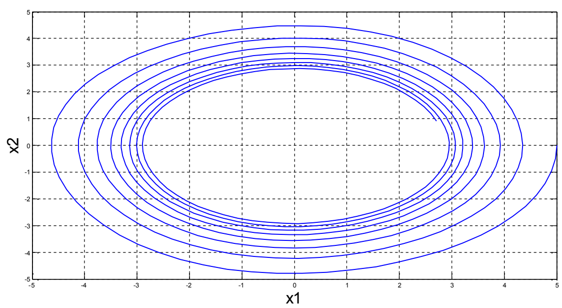

Fig 5.4. Limit cycle representation in Phase plane, For $x1$(0)=5, $x2$(0)=0 , ε=0.01.

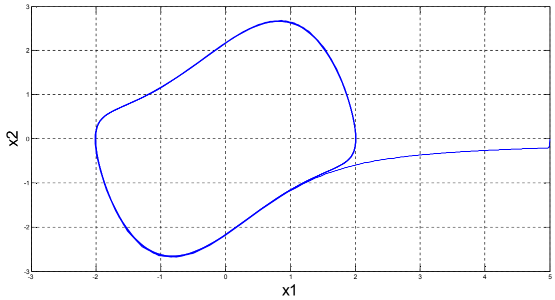

Fig 5.5. Limit cycle representation in Phase plane, For $x1(0)=5$, $x2(0)=0$, $\varepsilon=1$.

Initially the scalar parameter 'ε' is maintained as 0.1 and subsequently varied in both ways. The model is tested for a variety of initial conditions to observe its behaviour, and the limit cycle representations in phase plane obtained are depicted in Fig's 5.1 to 5.5. As inferred from the above figures, it is perceived that the model has a stable limit cycle in phase plane. When $x2$ is plotted against $x1$, it is observed that the system states approach a limit cycle approximating a circle at the origin with a radius of 2.

6. Analysis on Coupled Van Der Pol Oscillators

Two simulated Van der pol oscillators (VDP) are coupled by means of resistive coupling and the response of the coupled pair for various coupling strengths is analyzed with the scalar parameter 'ε' maintained as 0.1 throughout. On maintaining the coupling conductance (G_C) between the two van der pol oscillators at G_C=1 Siemen (S) the two oscillators fail to synchronize as seen in Fig 6.1. There exist much difference in both amplitude and phase in the simulated response.

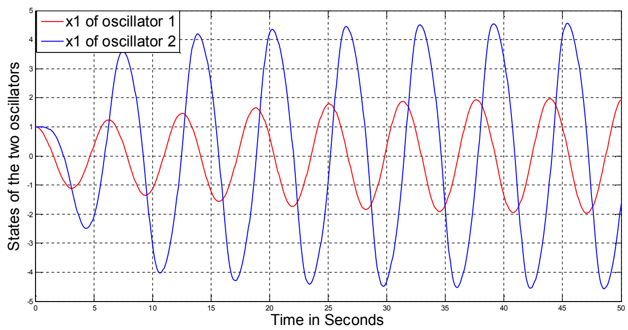

Fig 6.1. Response of Coupled VDP oscillators with G_C = 1S.

The G_C is further varied accordingly to ascertain if the coupled oscillators can attain synchrony. A reduction of the G_C value to 1/10S is seen to improve the system response where the amplitude and phase differences of the oscillator states decreases as before which is evident from Fig 6.2.

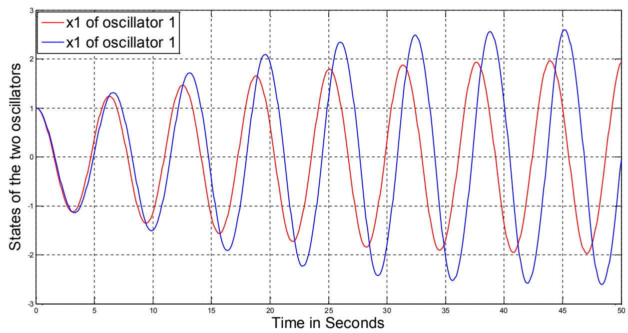

Fig 6.2. Response of Coupled VDP oscillators with G_C = 1/10S.

A further decrease in the G_C value to 1/100S yields a better result with negligible amplitude and phase difference among the states of the coupled oscillator pair. The response obtained is presented in Fig 6

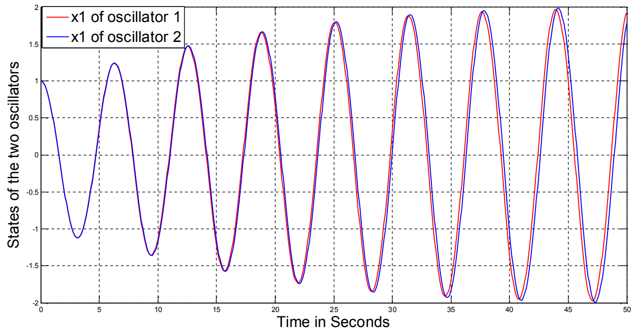

Fig 6.3. Response of Coupled VDP oscillators with G_C = 1/100S.

Eventually the lowest kept conductance value of 1/1000 S establish synchrony among the coupled oscillators as seen in the figure 6.4 where the states of the oscillators are in phase with one another.

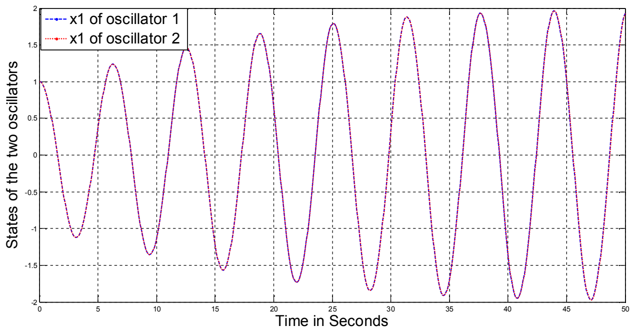

Fig 6.4. Response of Coupled VDP oscillators with G_C = 1/1000S.

From the simulated results, the coupling conductance of G_C=1/100S is considered throughout for further analysis, which yields a small phase shift between the states of the coupled pair of oscillators.

6.1 Analysis of parameter variations 'ε' in the oscillator model

While maintaining the parameter '$\varepsilon 1$' of the first oscillator at 0.1, the scalar parameter '$\varepsilon 2$' of the second oscillator is reduced by a factor of 10 (0.01) for which the states exhibit an amplitude difference and a small phase difference, eventually not in synchrony as seen in fig 6.5.

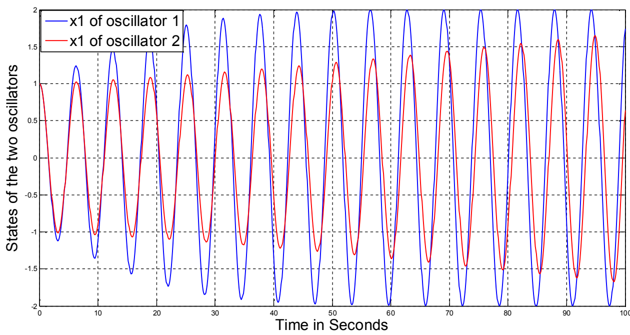

Fig 6.5. Response of state x1 for two Coupled VDP oscillators. $\varepsilon 1 > \varepsilon 2$.

Similarly when '$\varepsilon 2$' alone is increased to 1 while maintaining '$\varepsilon 1$' as before, the states of the coupled oscillators again fail to attain synchrony with differences in amplitude as well as phase which is seen from fig 6.6.

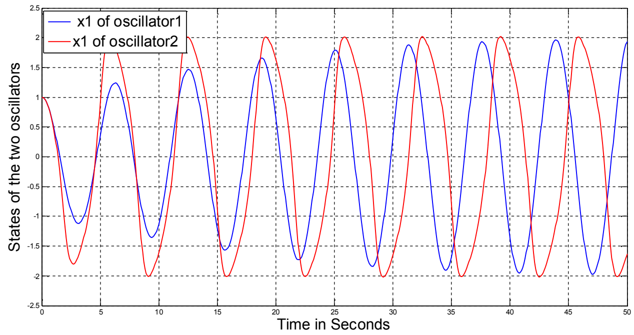

Fig 6.6. Response of state x1 for Coupled VDP oscillators. $\varepsilon 1 < \varepsilon 2$.

From the above simulated results it can be perceived that for any variations in the variable parameter 'ε', the state of the coupled oscillators fail to be in synchrony with appreciable amplitude and phase differences. Since the ultimate motto is to establish synchrony between the coupled oscillators both in amplitude and phase, different control techniques are proposed, and the same designed and implemented.

7 Controllers

This section briefly explains the theory on various control modes implemented to synchronize two coupled van der pol oscillators that are otherwise not in synchrony.

7.1 Proportional Control

The most basic of the continuous control modes usually referred by the letter 'P' which aims to control the process as the conditions change is proportional control. The larger the proportional band, the more stable the control, but the greater the offset. The narrower the proportional band, the less stable the process, but the smaller the offset in a proportional controller, steady state error tends to depend inversely upon the proportional gain, so if the gain is made larger, the error goes down [9]. The proportional response can be adjusted by multiplying the error 'e(t)' by a constant K_p, called the proportional gain constant. The proportional term is given by: $u(t) = k_p e(t)$ which is proportional to the error, hence the name proportional controller where '$u(t)$' is the controller output. A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is too high, the system can become unstable.

7.2 Proportional + Integral Control

The PI controller combines the behaviour of the I and P controllers, allowing the advantages of both controller types to be combined: fast reaction and compensation of remaining system deviation [10]. The Proportional with integral term is given by, $u(t) = k_p e(t) + k_i \int_0^t e(t)\,dt$. The integral term accelerates the movement of the process towards the set point and eliminates the residual steady-state error that occurs with a pure proportional controller. However, since the integral term responds to accumulated errors from the past, it can cause the present value to overshoot the set point value.

7.3 Proportional + Derivative Control

The PD controller consists of a combination of proportional action and differential action. The differential action describes the rate of change of the system deviation. The derivative-action time T_d is a measure for how much faster a PD controller compensates a change in the controlled variable than a pure P controller [11]. The magnitude of the contribution of the derivative term to the overall control action is termed the derivative gain, k_d. The proportional with derivative term is given by, $u(t) = k_p e(t) + k_d \frac{d}{dt} e(t)$. The derivative action predicts system behaviour and thus improves settling time and stability of the system but is seldom used in practice because of its inherent sensitivity to measurement noise. If this noise is severe enough, the derivative action will be erratic and actually degrade the control performance.

7.4 Proportional + Integral + Derivative Control

PID control is extremely common in industry, due to its ease of design however, it is worth noting that it may not be desirable to implement the controller, due to the derivative term. The functions of the PID controller includes providing feedback, the ability to eliminate steady state offsets through integral action and to anticipate the future through derivative action [12].Specifically, the PID controller differentiates the error, which is defined as error, e = r – y (r is the set point and y is the process variable). If r is a step input, there will be a discontinuity in e, and differentiating this would produce very large (theoretically infinite) signals. Thus, the proportional with integral and derivative term is given by, $u(t) = k_p e(t) + k_i \int_0^t e(t)\,dt + k_d \frac{d}{dt} e(t)$. The additional integral mode corrects for any offset (error) that may occur between the desired value (set point) and the process output automatically over time [13].

7.5 Fuzzy Logic Control

Fuzzy Logic (FL) is a problem-solving control system methodology that lends itself to implementation in systems ranging from simple, small, embedded micro-controllers to large, networked, multi-channel PC or workstation-based data acquisition and control systems. Fuzzy Logic refers to a large subject dealing with a set of methods to characterize and quantify uncertainty in engineering systems that arise from ambiguity, imprecision, fuzziness and lack of knowledge. Fuzzy logic is a reasoning system based on a foundation of fuzzy set theory, itself an extension of classical set theory, where set membership can be partial as opposed to all or none, as in the binary features of classical logic [14]. FL provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information. FL's approach to control problems mimics how a person would make decisions, only much faster.

Fuzzy Logic incorporates a simple, rule-based IF ‘X’ AND ‘Y’ THEN ‘Z’ approach to solving a control problem rather than attempting to model a system mathematically. The FL model is empirically-based, relying on an operator's experience rather than their technical understanding of the system. The purpose of control is to influence the behaviour of a system by changing an input or inputs to that system according to a rule or set of rules that model how the system operates [15]. The system being controlled may be mechanical, electrical, chemical or any combination of these.

The major drawback of the PID control system is that it usually assumes that the system being modelled is linear or at least behaves in some fashion that is a monotonic function. As the complexity of the system increases it becomes more difficult to formulate that mathematical model. Fuzzy control replaces the role of the mathematical model and replaces it with another that is build from a number of smaller rules that in general only describe a small section of the whole system that is, a fuzzy model has replaced the mathematical one [16]. There are different types of membership functions available, among which triangular membership function is used in this work.

8. Implementation of Control Strategies and Simulation results

The proposed control strategies are designed and implemented and the simulation results are presented below.

8.1 Proportional Controller

The controller parameters are estimated from the cost function where the value of the controller parameters that yields the lowest ISE (Integral Square Error) value is selected. The plot of the same for the case of a P controller is illustrated in Fig 8.1.1.

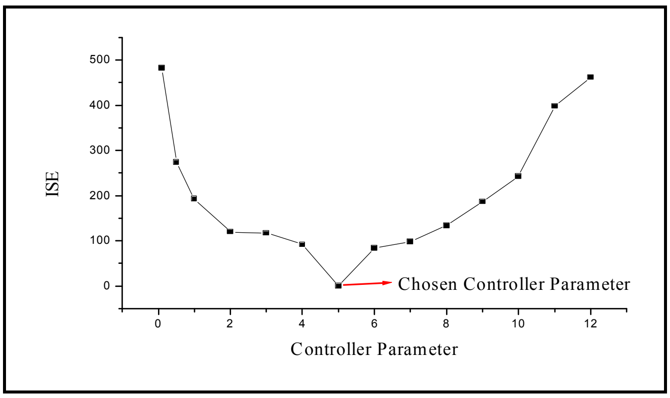

Fig 8.1.1. Cost function plot to obtain controller parameter for P Control.

With identical values of ε as 0.1 for both the coupled identical oscillators a Proportional Control scheme is implemented for which the state of the oscillators attain synchrony and the simulated response is shown in fig 8.1.2.

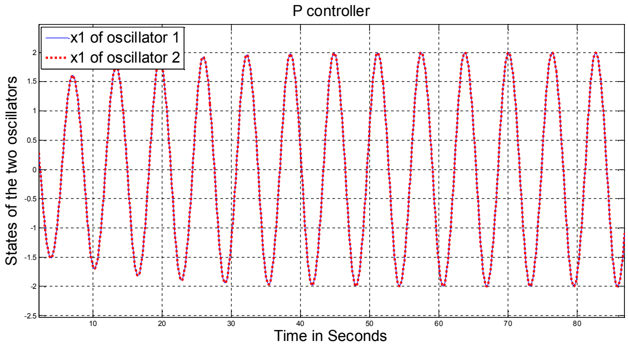

Fig 8.1.2. Response of coupled VDP oscillators in synchrony with P Control for $\varepsilon 1 = \varepsilon 2$.

A further increase made in the $\varepsilon 2$ of the second oscillator to 0.2 while maintaining $\varepsilon 1$ as before also synchronizes the coupled pair with P control, as shown in the fig 8.1.3.

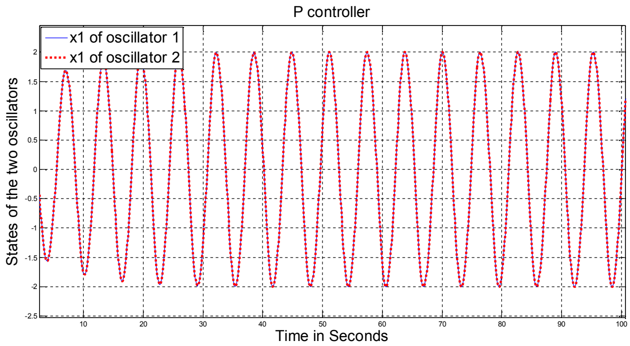

Fig 8.1.3. Response of coupled VDP oscillators in synchrony with P Control for $\varepsilon 1 < \varepsilon 2$.

The ε value of the second oscillator is reduced and P control implemented. For the parameters $\varepsilon 1$=0.1 and $\varepsilon 2$=0.01 the two oscillators eventually maintain synchrony as shown in the fig 8.1.4.

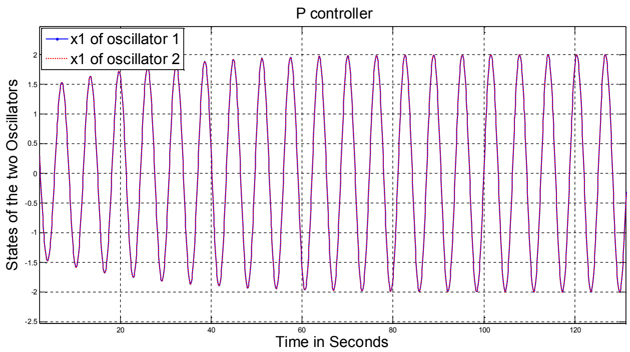

Fig 8.1.4 Response of coupled VDP oscillators in synchrony with P Control for $\varepsilon 1 > \varepsilon 2$.

8.2 Proportional + Integral Controller

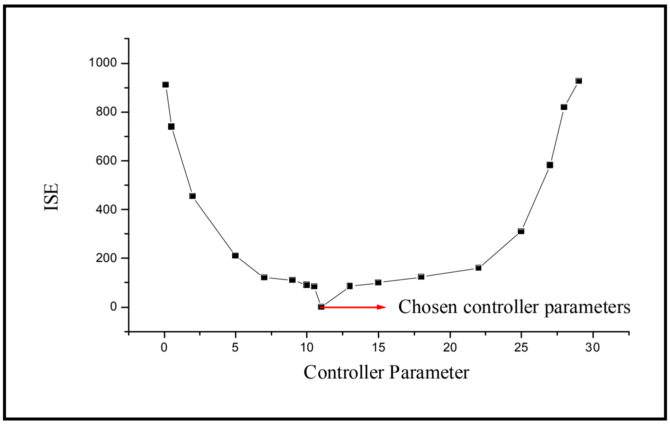

Fig 8.2.1. Cost function plot to obtain controller parameters for PI Control.

With identical ε values maintained at 0.1 as before a PI control implemented is seen to synchronize the two coupled oscillators and the simulated response is presented in fig 8.2.2.

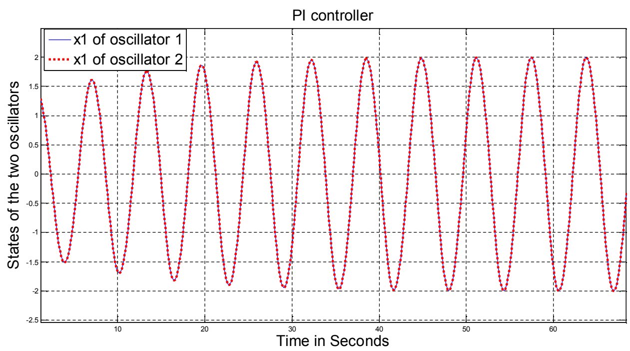

Fig 8.2.2. Response of coupled VDP oscillators in synchrony with PI Control for $\varepsilon 1 = \varepsilon 2$.

Upon maintaining $\varepsilon 1$ as before and $\varepsilon 2$ alone increased to 0.2 the implemented PI control scheme maintains synchrony of the coupled oscillator pair as shown in the fig 8.2.3.

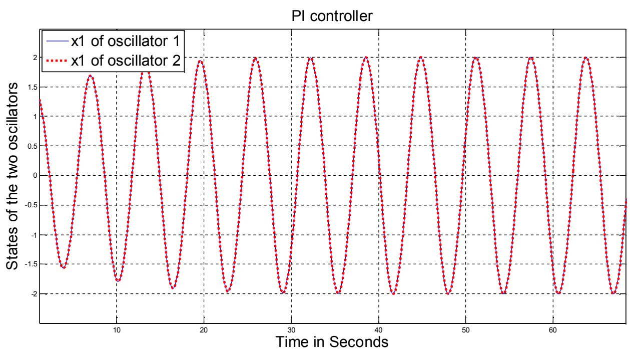

Fig 8.2.3. Response of coupled VDP oscillators in synchrony with PI Control for $\varepsilon 1 < \varepsilon 2$.

For a decreased $\varepsilon 2$ value as 0.01 for the second oscillator while maintaining $\varepsilon 1$, a P+I control implemented eventually maintain synchrony of the coupled oscillators as depicted in the response of fig 8.2.4.

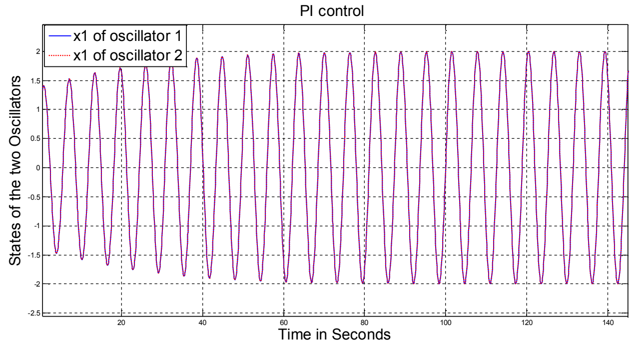

Fig 8.2.4. Response of coupled VDP oscillators in synchrony with PI Control for $\varepsilon 1 > \varepsilon 2$.

8.3 Proportional + Derivative Controller

With the aid of the controller parameters obtained from the cost function plot, it is tried to coax the coupled oscillators attain synchrony. For identical values of ε maintained at 0.1 for both the coupled oscillators the implemented PD Control scheme maintain synchrony between them as presented in the fig 8.3.2. The cost function plot for the desired PD control is presented in fig 8.3.1.

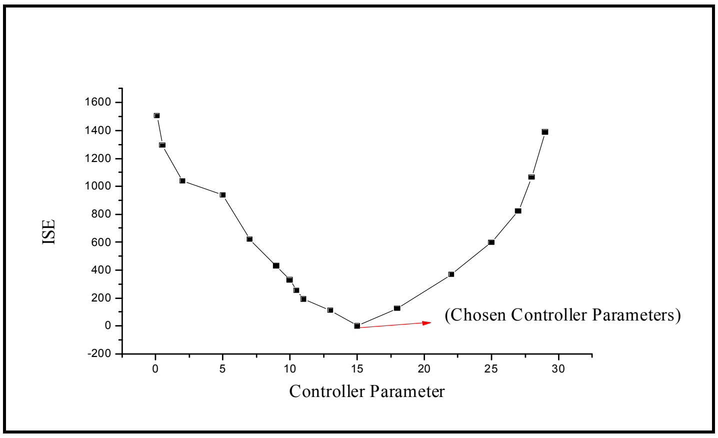

Fig 8.3.1. Cost function plot to attain controller parameters for PD Control.

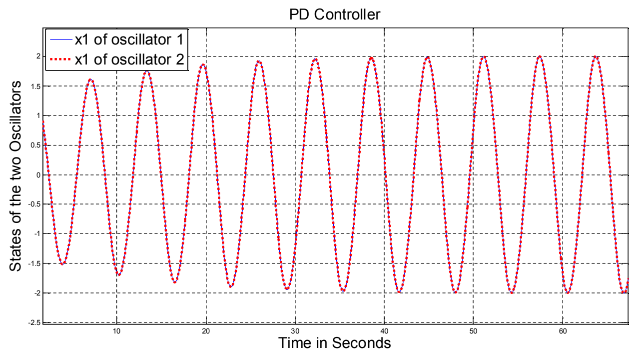

Fig 8.3.2. Response of coupled VDP oscillators in synchrony with PD Control for $\varepsilon 1 = \varepsilon 2$.

The parameter $\varepsilon 1$ is maintained as before while $\varepsilon 2$ is increased to 0.2 and the implemented PD control is seen to institute synchrony among the coupled oscillators as in fig 8.3.3.

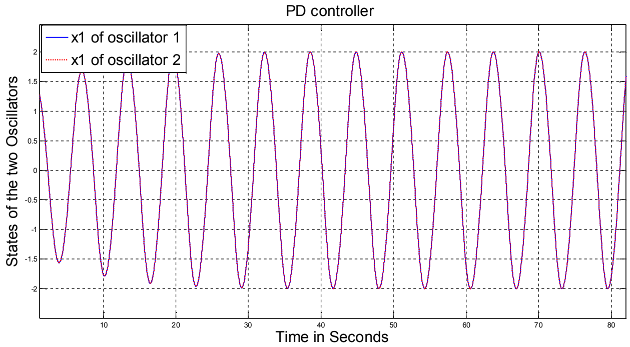

Fig 8.3.3. Response of coupled VDP oscillators in synchrony with PD Control for $\varepsilon 1 < \varepsilon 2$.

A decrease in the parameter $\varepsilon 2$ of the second oscillator alone to 0.01 while maintaining $\varepsilon 1$ of the first oscillator is made and the PD Control implemented. The control scheme maintains the coupled oscillators in synchrony as shown in the response of fig 8.3.4.

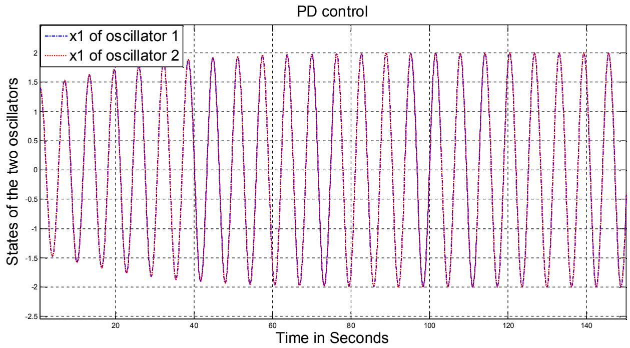

Fig 8.3.4. Response of coupled VDP oscillators in synchrony with PD Control for $\varepsilon 1 > \varepsilon 2$.

8.4 Proportional + Integral + Derivative Controller

The cost function plot obtained is again made use of to find the controller parameters for PID scheme as shown in fig 8.4.1.

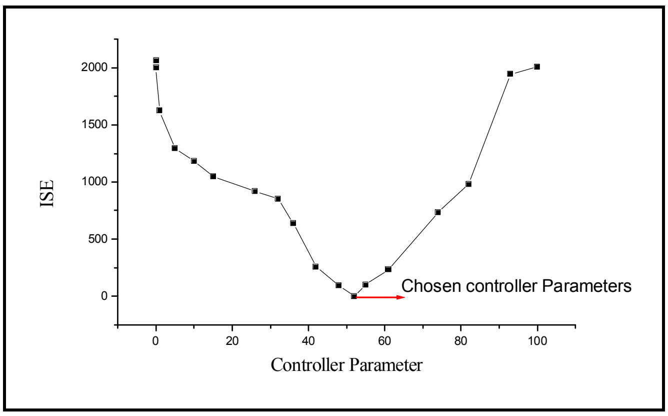

Fig 8.4.1. Cost function plot to obtain controller parameters for PID Controller.

For similar settings in the parameter 'ε' maintained at 0.1 for both the oscillators, the PID control implemented synchronizes the coupled pair and the response obtained is illustrated in fig 8.4.2.

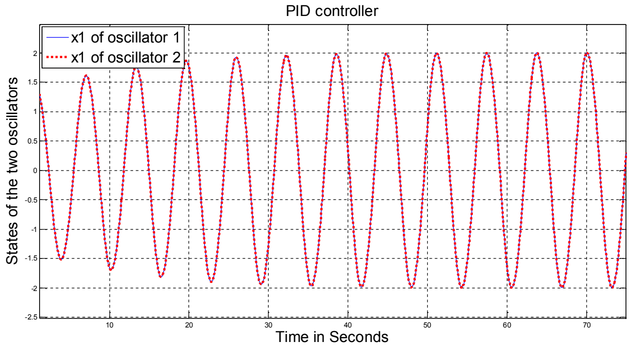

Fig 8.4.2. Response of coupled VDP oscillators in synchrony with PID Control for $\varepsilon 1 = \varepsilon 2$.

As before the parameter $\varepsilon 1$ is maintained, with $\varepsilon 2$ alone increased to 0.2 and PID control implemented. The control scheme synchronizes the two oscillators both in amplitude and phase and the simulated response is shown in the fig 8.4.3.

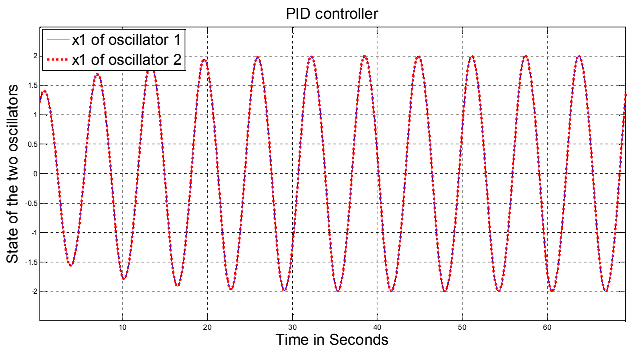

Fig 8.4.3. Response of coupled VDP oscillators in synchrony with PID Control for $\varepsilon 1 < \varepsilon 2$.

The parameter $\varepsilon 2$ of the second oscillator alone is now reduced to 0.01 while maintaining $\varepsilon 1$, and the implemented PID control synchronizes the coupled oscillators as seen in fig 8.4.4.

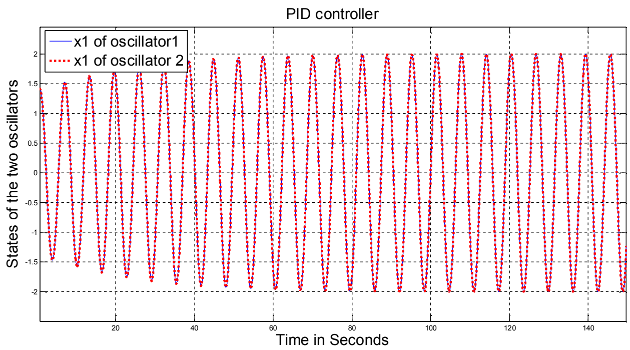

Fig 8.4.4. Response of coupled VDP oscillators in synchrony with PID Control for $\varepsilon 1 > \varepsilon 2$.

8.5 Fuzzy Logic Controller

A Mamdani type fuzzy logic control (FLC) is designed with triangular membership function with the appropriate rule base as seen in the rule viewer of fig 8.5.1.

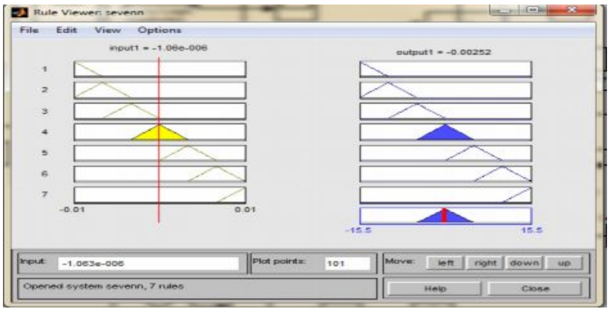

Fig 8.5.1. Rule viewer in the implemented Fuzzy logic control.

All the rules that apply are invoked, using the membership functions and truth values obtained from the inputs, to determine the result of the rule which in turn will be mapped into a membership function and truth value controlling the output variable. The results are combined to give a specific ("crisp") answer, a procedure known as "defuzzification". These rules are typical for controlling in that, the antecedents consist of the logical combination of the error signals and the consequent is a control command output [17].

The implemented FLC maintains synchrony for the coupled oscillators for identical parameters of 'ε' maintained at 0.1as shown in the fig 8.5.2.

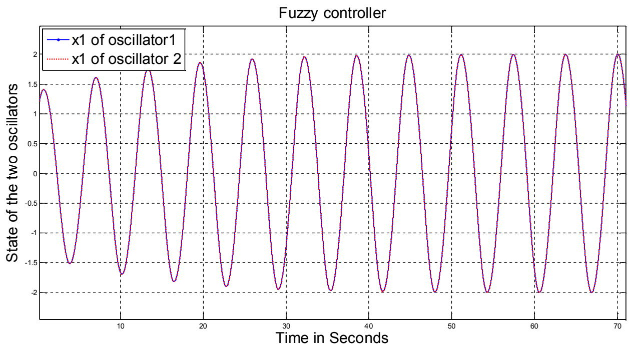

Fig 8.5.2. Response of coupled VDP oscillators in synchrony with FLC for $\varepsilon 1 = \varepsilon 2$.

The parameter '$\varepsilon 1$' is maintained as before with '$\varepsilon 2$' alone increased to 0.2 and FLC scheme implemented. The FLC control eventually synchronizes the two coupled oscillators both in amplitude and phase as shown in fig 8.5.3.

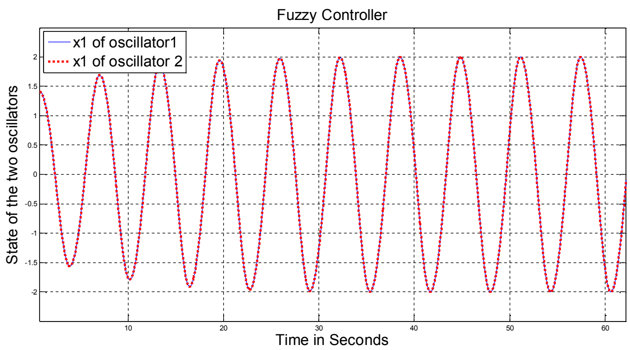

Fig 8.5.3. Response of coupled VDP oscillators in synchrony with FLC for $\varepsilon 1 < \varepsilon 2$.

The parameter '$\varepsilon 2$' is now decreased to 0.01 by maintaining '$\varepsilon 1$' same as before, and the FLC scheme implemented. The FLC maintain synchrony among the coupled oscillators as shown in fig 8.5.4.

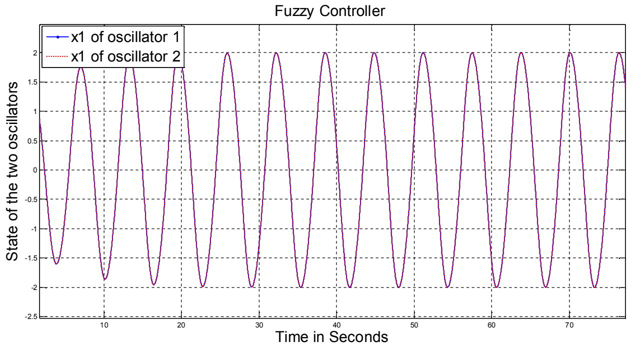

Fig 8.5.4. Response of coupled VDP oscillators in synchrony with FLC for $\varepsilon 1 > \varepsilon 2$.

8.6 Performance Evaluation

The problem of designing the 'best' controller can be formulated as the type of the controller and the values of its adjusted parameters so as to minimize the (Integral of the Square Error) ISE of the system's response [12]. To strongly suppress large errors, ISE is better because the errors are squared and thus contribute more to the value of the integral and hence the same is opted in this work where,

$$ISE = \int_0^{\infty} E^2(t)\,dt$$

and E(t) is the error. In addition to ISE, steady state errors as well as the synchronization time(both in amplitude and phase) are considered and the same tabulated for all variations effected.

Table 1: ε1= ε2.

S.No	Controller	ε1	ε2	$G_C(S)$	ISE	S.S. Error	Synchronization time (Sec)
1	P	0.1	0.1	1/100	0.00099	−0.0004432	X=65 V=60
2	PI	0.1	0.1	1/100	0.001007	−0.0003481	X=65 V=65
3	PD	0.1	0.1	1/100	0.00001926	−0.0002803	X=65 V=50
4	PID	0.1	0.1	1/100	0.00001542	−0.0002411	X=65 V=65
5	FUZZY	0.1	0.1	1/100	0.00000465	−0.0001093	X=60 V=60

Table 2: ε1<ε2.

S.No	Controller	ε1	ε2	$G_C(S)$	ISE	S.S. Error	Synchronization time (Sec)
1	P	0.1	0.2	1/100	0.06676	−0.002015	X=45 V=45
2	PI	0.1	0.2	1/100	0.06681	−0.00187	X=50 V=45
3	PD	0.1	0.2	1/100	0.0001229	−0.0008279	X=45 V=45
4	PID	0.1	0.2	1/100	0.00003118	−0.0004099	X=45 V=40
5	FUZZY	0.1	0.2	1/100	0.000003218	−0.0001053	X=45 V=40

Table 3: ε1>ε2

S.No	Control	ε1	ε2	G_C (S)	ISE	S.S. Error	Synchronization time (Sec)
1	P	0.1	0.01	1/100	0.01591	−0.008019	X=120 V=135
2	PI	0.1	0.01	1/100	0.01595	−0.008021	X=120 V=135
3	PD	0.1	0.01	1/100	0.0009434	−0.000535	X=120 V=125
4	PID	0.1	0.01	1/100	0.0000434	−0.0000883	X=120 V=125
5	FUZZY	0.1	0.01	1/100	0.0000135	−0.0000413	X=35 V=40

From the performance evaluation tables presented above it is observed that Fuzzy controller exhibits better performance with the lowest ISE as well as the steady state error. FLC exhibits faster response, consuming lesser time to attain synchronization both in amplitude and phase of the response well ahead of the rest.

9. Conclusion

When two oscillators operate physically near one another the output from either one of them can influence the behavior of the other. The coupled oscillators are observed to be phase locked since they exhibit a constant phase difference regardless of being small, medium or large. Two Van der pol Oscillators are coupled and the phenomena of synchronization is studied using MATLAB Software. Five different control strategies for a pair of coupled van der pol oscillators that are otherwise not in synchrony have been designed and the performance measures of the implemented control schemes evaluated.

Out of the various control schemes to entrain two van der pol oscillator to synchronize over time, the simulation results quantify that the proposed Fuzzy Logic Control (FLC) scheme gives optimal performance over the other control strategies. Interacting nonlinear oscillators with different individual frequencies can spontaneously synchronize themselves to a common frequency only if the coupling strength exceeds a certain threshold value. This phenomenon is of much relevance to the understanding of biological oscillators such as the coupled heart pacemaker cells [18]. The mutual interactions between the two coupled van der pol oscillators are found to be phase dependent and can have either an advancing or delaying influence on the other oscillators discharge so that, equality of period i.e., mutual entrainment within the Sino Atrial (SA) node of the cardiac system can be the end result.

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